Appendix O:
Solving a System of Equations in Two Variables by Graphing

TERMINOLOGY

For each of the following terms, provide 1) a definition in your own words, 2) the formal definition (as provided by your text or instructor), and 3) an example of the term using a drawing or problem. A sample filled-out form is available in the Introduction.

System of Equations

Your definition	
Formal definition	
Example	

Solution of a System

Your definition	
Formal definition	
Example	

Consistent System

Your definition	
Formal definition	
Example	

Inconsistent System

Your definition	
Formal definition	
Example	

Dependent System

Your definition	
Formal definition	
Example	

READING AND SELF-DISCOVERY QUESTIONS — APPENDIX O

1. What happens when two equations which have different slopes are graphed on the rectangular coordinate system?

   The two equations, when graphed, will eventually intersect at a point.

2. When lines intersect at a unique point, what is that point called? How is it represented mathematically (not graphically)?

   The point is called the point of intersection. It represents the solution to the system of equations represented by the two graphs.

3. If two lines do not intersect, what do we know about the lines? Is there a solution to the system of equations represented by the lines? Why or why not?

   If two lines do not intersect, then the lines are parallel. There is no solution to the system of equations because solutions are represented graphically by points of intersection.

CRITICAL THINKING QUESTIONS — APPENDIX O

1. What does a solution to a system of linear equations look like graphically? Mathematically? (Be sure to explain, not simply provide examples.)

   Graphically, the solution to a system of linear equations looks like a single point of intersection, no point of intersection (if the lines are parallel), or an infinite number of points (if the lines are the same line).
   Mathematically, the solution looks like $x = 2$, $y = 3$, no solution, or an infinite number of solutions.

2. In a system of linear equations, how can the slopes of the equations give you a clue as to what kind of solution you will find?

   If the slopes are different, then there will be one solution. If the slopes are the same, then there are no solutions or an infinite number of solutions.

3. Why is the ability to find a solution to a system of linear equations an important skill? In your answer, provide at least one real-world example where this skill is applied.

   Student answers will vary. A sample answer might include: The break even point for making a profit in manufacturing and selling a product is the point at which two linear equations (the cost of manufacturing the product is one, while the amount the product sells for is the other) intersect.

4. Can you ever have a system of linear equations that has precisely four solutions? Why or why not?

   No. Systems can only have one, none, or an infinite number of solutions.

DEMONSTRATE YOUR UNDERSTANDING — APPENDIX O

1. Write a system of linear equations where the solution is (3, 2).

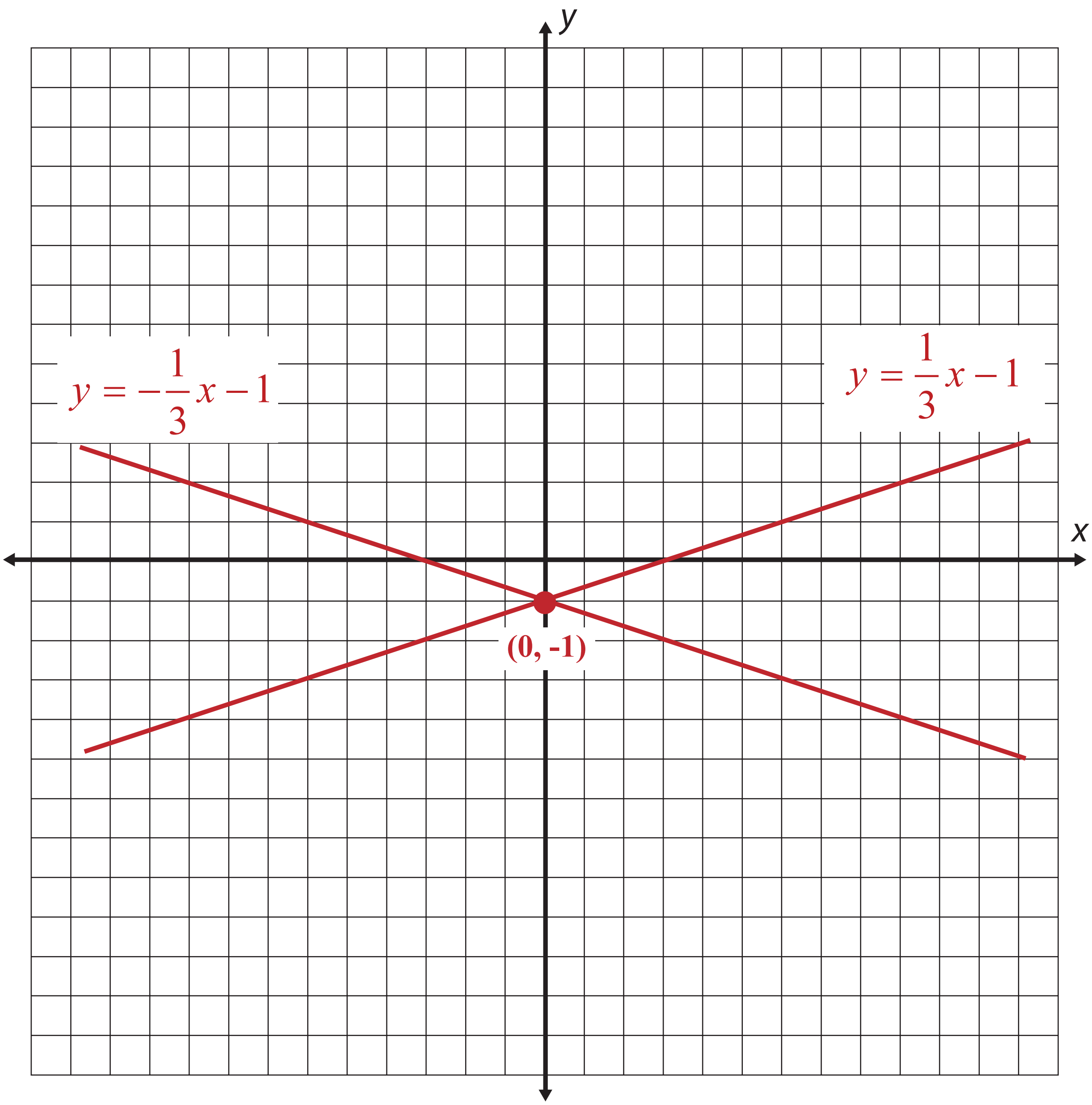
   A system of linear equations with solution in coordinate form is $\begin{cases} x - 3y = -3 \\ 2x + 4y = 14 \end{cases}$

   This is only a sample answer; student answers will vary.

2. Solve by graphing: $\frac{1}{3}x + y = -1, \quad x - 3y = 3$

$\frac{1}{3}x + y = -1$ in slope-intercept form: $y = -\frac{1}{3}x - 1$

$x - 3y = 3$ in slope-intercept form: $y = \frac{1}{3}x - 1$



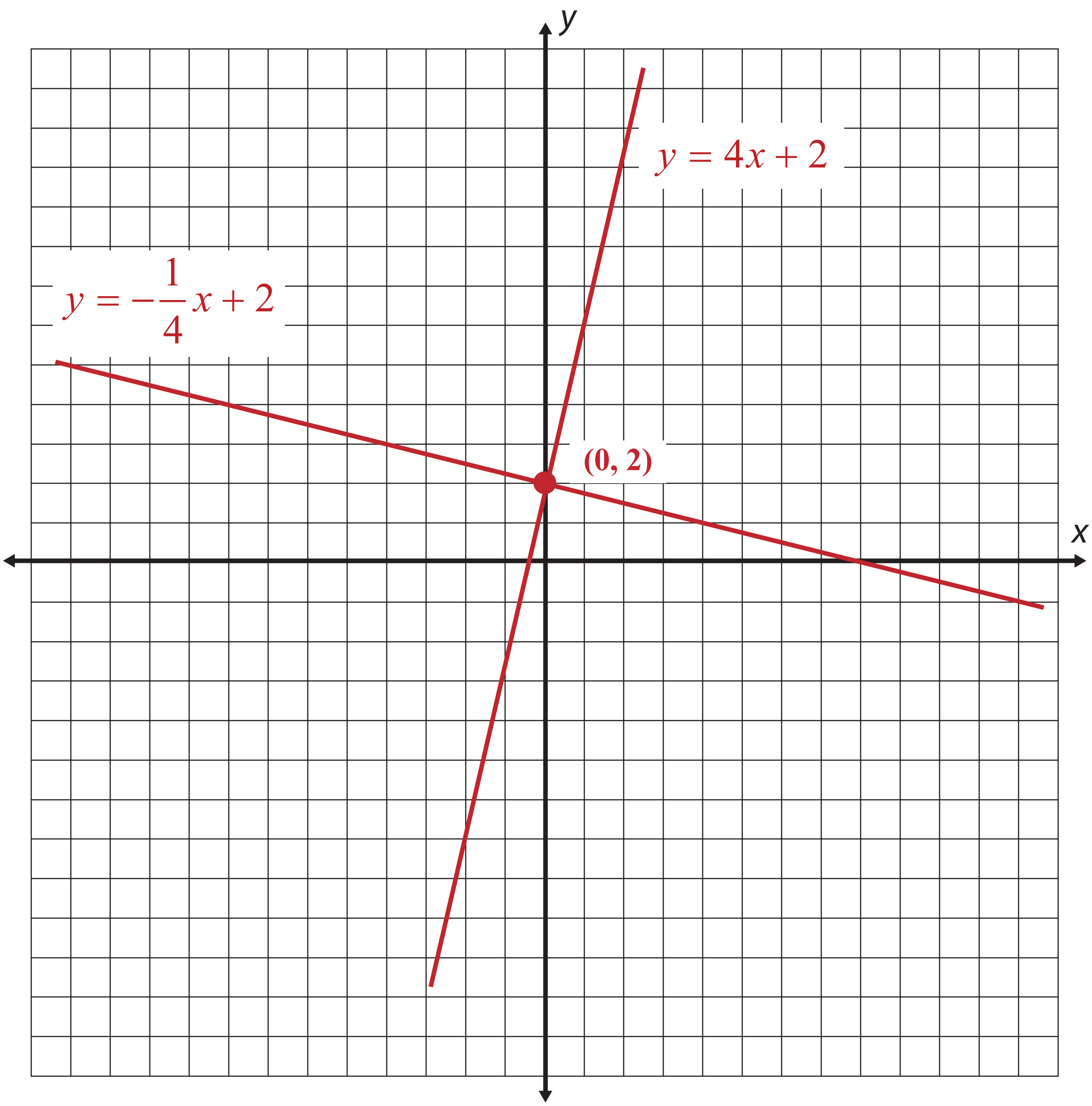
The solution is obvious if we notice that the y-intercept for both equations is –1, once they are put into point-slope format.

Solution:

(0, –1)

3. Solve by graphing: $x+4y=8, \quad y=4x+2$

$x+4y=8$ in slope-intercept form: $y=-\frac{1}{4}x+2$



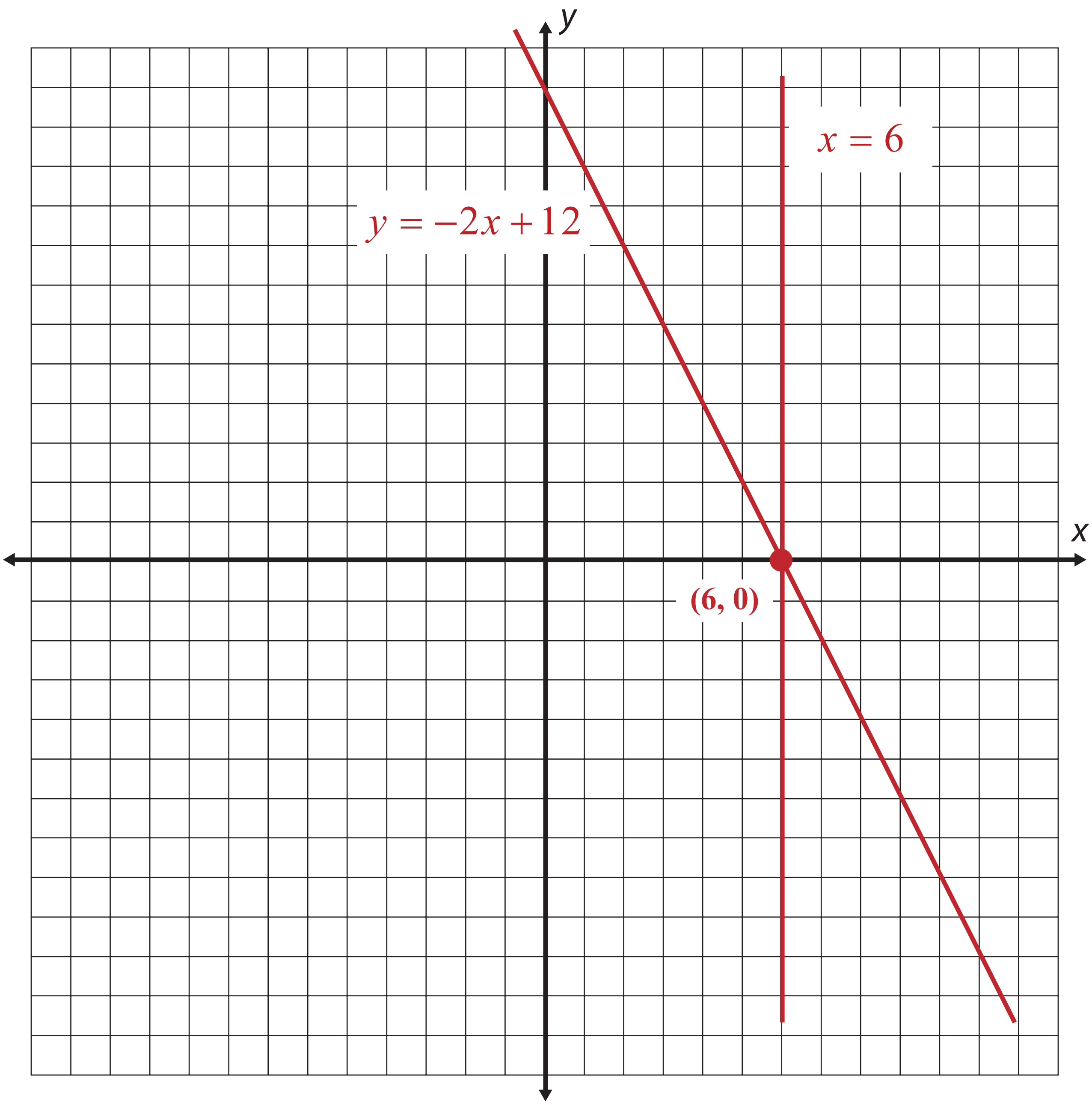
The solution is obvious if we notice that the y-intercept for both equations is 2.

Solution:

(0, 2)

4. Solve by graphing: $2x + y = 12$, $-\frac{1}{2}x + 6 = 3$ $2x + y = 12$ in slope-intercept form: $y = -2x + 12$

$-\frac{1}{2}x + 6 = 3$ is, after combining terms: $x = 6$



Solution:

(6,0)