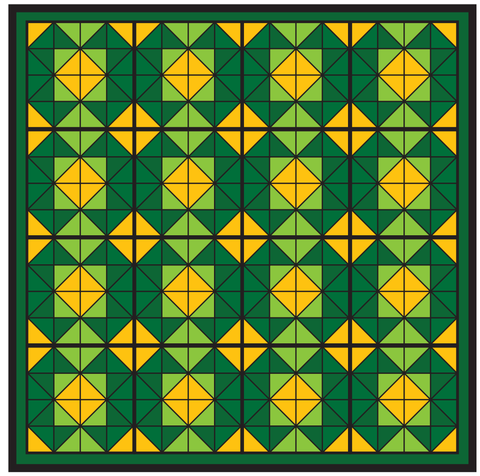
Section 3.6

PRE-ACTIVITY PREPARATION

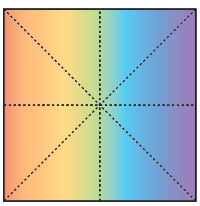
Composite Figures

Leisure activities often include the use of different combinations of basic shapes. Below are some examples of how we might use basic shapes in complex patterns that are useful or pleasing to the eye or in challenging intellectual activities.

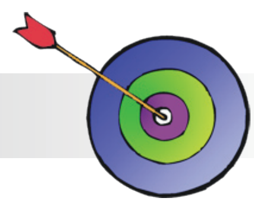


Tangrams—a set of tiles consisting of seven geometric shapes: five triangles, a square and a parallelogram. The player is to arrange the tiles into specified figures or shapes. Many mathematical principles have been applied to the use of the tiles and it has been determined that there are only 13 closed pattern shapes. On the other hand, whimsical open shapes and patterns are essentially endless.

Quilting—Traditional quilt patterns use combinations of squares and triangles to build bigger squares and shapes. The quilt at right is made from colorful squares and triangles and has a unique geometric design.



Origami—The Japanese art form of paper folding is closely tied with geometry. Any basic fold has an associated geometric pattern. For instance, when you fold the traditional water bomb base, you have created a crease pattern with eight congruent right triangles.



LEARNING OBJECTIVES

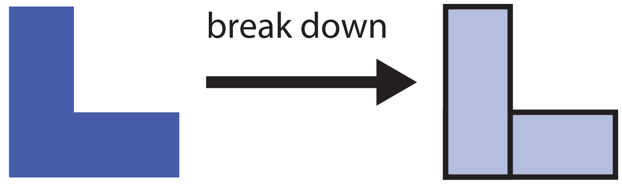
- Find the area and perimeter of irregularly shaped geometric figures
- Learn how to recognize basic shapes within more complex figures
- Add or subtract basic shapes to find the area and perimeter of more complex figures
- Use algebraic concepts to scale geometric figures

TERMINOLOGY

PREVIOUSLY USED	NEW TERMS TO LEARN
area	**augment**
perimeter	**composite figure**
radius	**polygon**
simplify	**regular polygon**
	scale

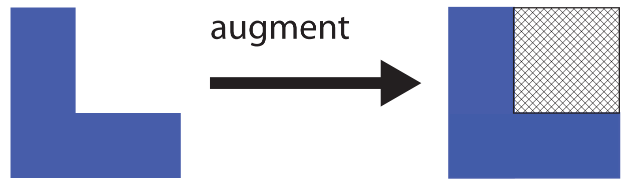
Building Mathematical Language

Breaking down a Drawing



Learning how to break a complex task down into simpler components is a skill that can transfer to any complicated problem. In geometry it is often necessary to break apart a drawing into basic shapes so that the **area** or **perimeter** can be calculated. At right is a geometric object that can be divided into two small rectangles. You can think of the figure as being *composed* of two rectangular shapes; therefore it is a **composite figure**.

Augmenting a Drawing



The same figure can also be **augmented** into one large rectangle.

Calculations with Composite Figures

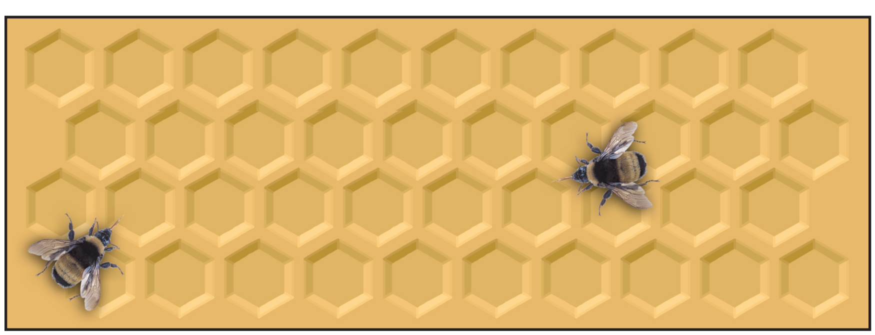
To calculate the area, we can either add the two smaller areas created by breaking down the drawing or subtract the hash marked area from the larger augmented rectangle area.

Polygons

A closed geometric figure is called a **polygon** and is classified by the number of sides it has.

Figure	Number of Sides	Example
Triangle	3	school pennant
Quadrilateral	4	football field
Pentagon	5	Pentagon Building
Hexagon	6	one cell in a honeycomb
Heptagon (Septagon)	7	50 pence coin (England)
Octagon	8	stop sign
Polygon	many sided	

The list can continue with Greek or Latin prefixes indicating the number of sides and the suffix *-gon*. A figure with many sides is simply called a **polygon**. If the sides are of equal length, then it is referred to as a **regular polygon**.

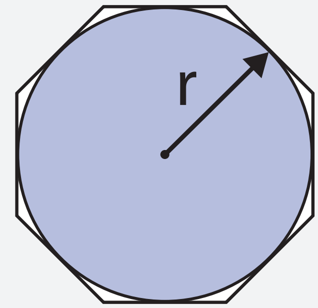


Area of a Regular Polygon

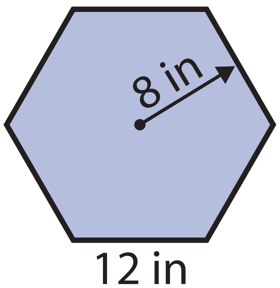
The area of a regular polygon can be found using the perimeter (P) and the distance from the center to the midpoint of a side (r).

$$A = \tfrac{1}{2} r P$$

We use ***r*** because of the relationship between the polygon and its inscribed circle—a circle inside the figure that touches each side at its midpoint.



For example, the area of a regular hexagon with sides of 12 in and a length of 8 in from the center to the side is:



$P = 6 \times 12$ in

$P = 72$ in

$A = \frac{1}{2}(8 \text{ in})(72 \text{ in})$

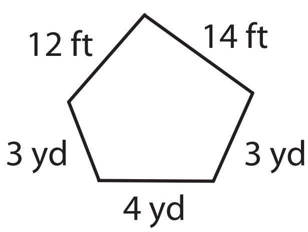
$A = (4 \text{ in})(72 \text{ in})$

$A = 288 \text{ in}^2$

Models

Model 1

Find the perimeter in feet.



Change yards to feet: $\frac{3 \text{ yd}}{x \text{ ft}} = \frac{1 \text{ yd}}{3 \text{ ft}}, \ x = 9 \text{ ft}$

$\frac{4 \text{ yd}}{x \text{ ft}} = \frac{1 \text{ yd}}{3 \text{ ft}}, \ x = 12 \text{ ft}$

$P = 12 \text{ ft} + 14 \text{ ft} + 9 \text{ ft} + 12 \text{ ft} + 9 \text{ ft}$

Answer: **P = 56 ft**

The Pentagon in Arlington, Virginia

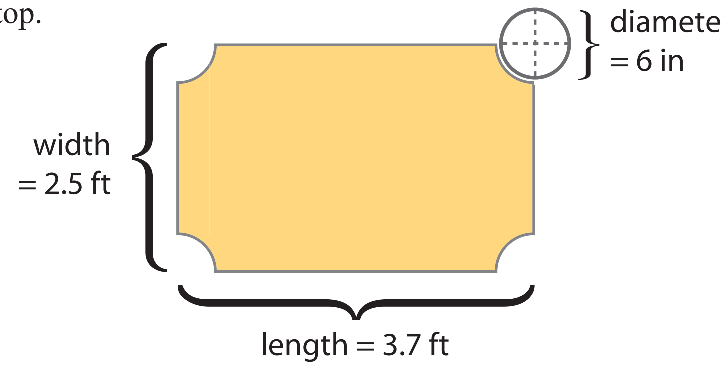
(Each side is 921 feet long; what is the perimeter?)

Model 2

Find the approximate area of the following table top.

REASONING

The figure is a rectangle with four decorative cutouts that are each 1/4 of a circle. Find the area of the rectangular table and then subtract the cutout sections (4 × 1/4 of a circle or one whole circle).



Augmented table:

$A = (3.7\text{ ft})(2.5\text{ ft}) = 9.25\text{ ft}^2$

Four quarter-circle cutouts (one whole circle):

$$r = \frac{d}{2} = \frac{6\text{ in}}{2} = 3\text{ in}$$

(Convert to feet: r = 3 in = 0.25 ft)

$A = \pi r^2$

$A \approx 3.14(0.25\text{ ft})^2$

$A \approx 0.2\text{ ft}^2$

Approximate area of the table:

$9.25\text{ ft}^2 - 0.2\text{ ft}^2 = \mathbf{9.05\text{ ft}^2}$

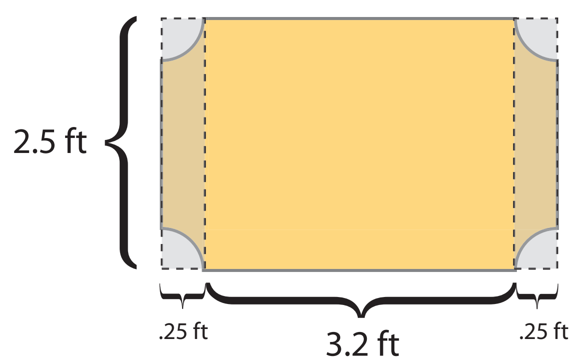
A good way to validate calculations with composite figures is to use an alternative method, when possible. Model 3 presents one such alternative way of solving this same problem.

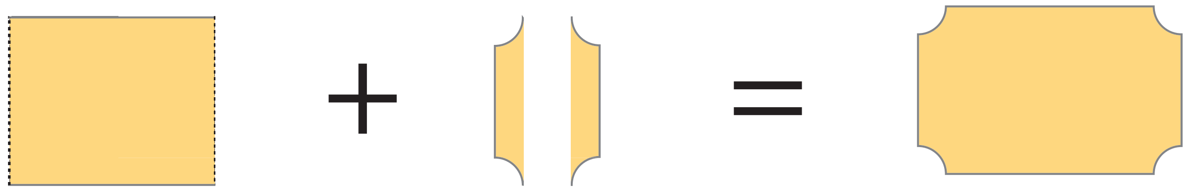
Model 3

Find the approximate area of the following table top.

REASONING

The figure is a rectangle with two decorative ends. Each decorative end is a rectangle with two 1/4 circle cutouts. Find the area of the table (the rectangle without the decorative ends). Then find the area of two decorative ends (each a rectangle minus a half-circle). Finally, add the area of the center portion of the table to the area of the two decorative ends.

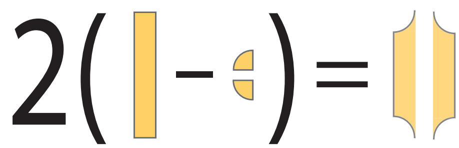




Center table section:

$A = 2.5 \text{ ft} \times 3.2 \text{ ft} = 8 \text{ ft}^2$

Two decorative ends:



$A \approx 2(2.5 \text{ ft} \times 0.25 \text{ ft} - 1/2(0.2 \text{ ft}^2))$

$A \approx 2(0.625 \text{ ft}^2 - 0.1 \text{ ft}^2)$

$A \approx 2(0.525 \text{ ft}^2)$

$A \approx 1.05 \text{ ft}^2$

area of the circle, as determined in Model 2

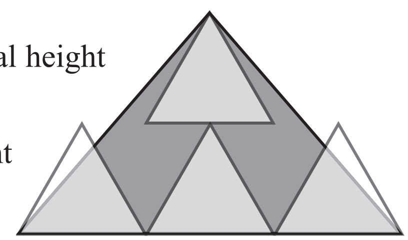
Approximate area of the table:

$8 \text{ ft}^2 + 1.05 \text{ ft}^2 = \mathbf{9.05 \text{ ft}^2}$

Model 4: Scaling

How is the area of a triangle represented in algebraic terms, if the original height is doubled and the base is three times the original base?

Note: This is referred to as "**scaling** up." (If we were to halve the height and reduce the base to one third the original dimension, it would be "scaling down.")



REASONING

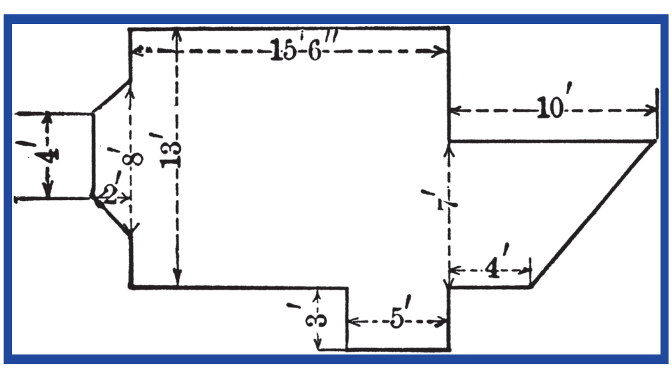
The basic formula for area of a triangle is: $\boldsymbol{A = ½\, bh}$. The area of the new (larger) triangle may be found by letting the base be $3b$ (three times the original base) and the height $2h$ (height is doubled).

The new area:

$A = ½(3b)(2h)$

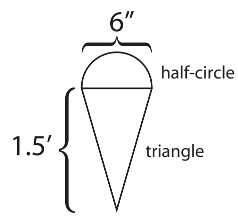
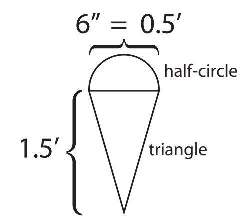
$A = ½ \times 3 \times 2 \times bh$

$\boldsymbol{A = 3bh}$



Floorplans provide a common example of composite figures. When you know the critical measurements of a room, space, or even an entire house, determining the total area is simply a matter of calculating individual areas and adding them together.

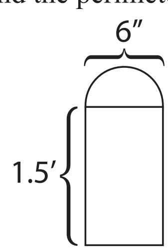
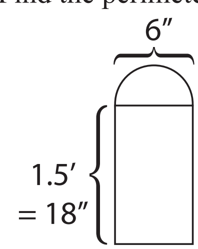
Addressing Common Errors

<table>
<tr><th>Issue</th><th>Incorrect Process</th><th>Resolution</th><th>Correct Process</th></tr>
<tr>
<td rowspan="3">Not converting to common units</td>
<td>Find the area:

$A = ½(6 \times 1.5) + ½(\pi 3^2)$
$A = ½ \times 9 + 4.5\pi$
$= 4.5 + 4.5\pi$
$= 18.63$</td>
<td>Always include the units with the calculations. When you try to add or multiply inches and feet, for example, you will be reminded to change from one to the other in order for the answer to be in the proper linear or square units.</td>
<td>Find the area:

$A = ½(0.5' \times 1.5') + ½(\pi(0.25')^2)$
$A = ½ \times 0.75\ \text{ft}^2 + ½ \times 0.0625\pi\ \text{ft}^2$
$\approx 0.375\ \text{ft}^2 + 0.098\ \text{ft}^2$
$\approx 0.473\ \text{ft}^2$</td>
</tr>
<tr><td colspan="3">Validation</td></tr>
<tr><td colspan="3">

- area units are square feet ✓
- convert 1.5 feet to 18 inches and solve the problem in inches:

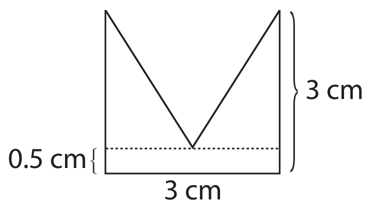
$A = ½(6'' \times 18'') + ½(\pi(3'')^2)$
$A = ½ \times 108\ \text{in}^2 + ½ \times 9\pi\ \text{in}^2$
$\approx 54\ \text{in}^2 + 13.18\ \text{in}^2$
$\approx 68.13\ \text{in}^2$

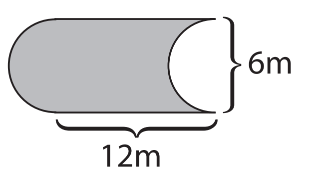
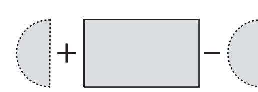
12 in × 12 in = 1 ft^2 = 144 in^2
Divide the answer in square inches by 144 and compare to the answer in square feet:

$\frac{68.13}{144} \approx 0.473\ \text{ft}^2$ ✓
</td></tr>
<tr><th>Issue</th><th>Incorrect Process</th><th>Resolution</th><th>Correct Process</th></tr>
<tr>
<td rowspan="3">Using the wrong formula</td>
<td>Find the perimeter:

$6'' = 0.5'$
$P = 2(w \times l) + \frac{1}{2}\pi r^2$
$P = 2(0.5' \times 1.5') + ½(\pi(0.25\ \text{ft})^2)$
$= 1.5 + 0.03125\pi$
≈ 1.6 ft</td>
<td>Carefully choose the correct formula, connecting the idea of area with square units and the words perimeter and circumference with length.</td>
<td>Find the perimeter:

Reason: sum of 3 sides + 1/2 circumference:
$P = 6'' + 2 \times 18'' + ½\pi(6'')$
$= 6'' + 36'' + 3\pi''$
≈ 51.42 in</td>
</tr>
<tr><td colspan="3">Validation</td></tr>
<tr><td colspan="3">

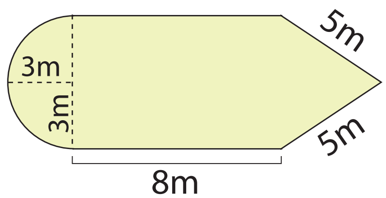
- perimeter units are linear ✓
- $6'' = 0.5'$
$P = 0.5' + 2 \times 1.5' + ½\pi(0.5')$
$\approx 0.5' + 3' + 0.785' \approx 4.285$ ft

$4.285 \times 12\frac{\text{in}}{\text{ft}} \approx 51.42$ in ✓
</td></tr>
</table>

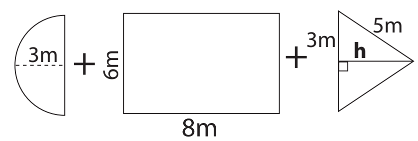
Issue	Incorrect Process	Resolution	Correct Process
Improper reasoning	Find the area of the figure below.  Area of square: $3^2 = 9\text{ cm}^2$ Area of rectangle: $(3)(0.5) = 1.5\text{ cm}^2$ Area of triangle: $\frac{1}{2}(3)(3 - 0.5) = 3.75\text{ cm}^2$ Answer: $9 + 1.5 - 3.75 = 6.75\text{ cm}^2$	With more complex composite figures, it is sometimes easy to lose track of what you are trying to solve. Consider shading in areas you have already worked or creating a list of the 'pieces' you are working with. Be sure to check the shaded areas or pieces against the measurements so you do not accidentally use the same area twice!	While the dimensions of the small rectangle are given, it is clearly part of the larger square. We are ultimately trying to solve for the 3 × 3 square minus the area of the triangle. The 0.5 cm measurement (and the dotted line) are there to help us figure out the height of the triangle.  Area of square: $3^2 = 9\text{ cm}^2$ Area of triangle: $\frac{1}{2}(3)(3 - 0.5) = 3.75\text{ cm}^2$ $\mathbf{9 - 3.75 = 5.25\text{ cm}^2}$
	Validation		
	• area units are square centimeters ✓ • we can break down the figure in an alternate way and solve:  Area of triangles: $2[\frac{1}{2}(1.5)(2.5)] = 3.75\text{ cm}^2$ Area of rectangle: $(0.5)(3) = 1.5\text{ cm}^2$ $\mathbf{3.75 + 1.5 = 5.25\text{ cm}^2}$ ✓		

Issue	Incorrect Process	Resolution	Correct Process
Making assumptions	Find the area:  $A = \frac{1}{2}$ cylinder $A = \frac{1}{2}(2B + Ch)$ $= \frac{1}{2}(2\pi r^2 + \pi dh)$ $= \frac{1}{2}(2\pi(3\text{ m})^2 + \pi 6\text{ m} \times 12\text{ m})$ $= \frac{1}{2}(18\pi + 72\pi)\text{m}^2$ $= 45\pi\text{ m}^2$	It is sometimes very tempting to assume information that is not included in the problem. Read problem statements and view any diagrams closely and carefully, making a note of exactly what information you have (and have not) been given.	The figure is a flat surface; there are no indications that it is a cylinder. The only assumption that can be made through examining the figure is that both ends are circular, with one end canceling out the other end.  $A = \frac{1}{2}\pi r^2 + (l \times w) - \frac{1}{2}\pi r^2$ $A = l \times w$ $A = (12m)(6m) = 72\ m^2$
	Validation		
	• area units are square meters ✓		

<table>
<tr><th>Issue</th><th>Incorrect Process</th><th>Resolution</th><th>Correct Process</th></tr>
<tr>
<td>Not visualizing a workable basic shape</td>
<td>What is the area of the following figure?



Dividing the figure with a horizontal line still doesn’t give the total length. Not enough information to solve.</td>
<td>You may have to try several different strategies to augment or break down a figure into parts you can work with.</td>
<td>In this case, we have a half-circle, a rectangle and a triangle. Because we have measurement for the base and sides of the triangle we can apply the Pythagorean Theorem to find the height of the triangle. That allows us to determine the area of the triangle. That, added to the area of the half-circle and the area of the rectangle will give us the area of the full figure.



Area of half-circle:

$A = \frac{1}{2}\pi(3)^2 = 4.5\pi$

$A \approx 14.13 \text{ m}^2$

Area of rectangle:

$A = lw = (6)(8) = 48 \text{ m}^2$

Height of triangle:

$h^2 = 5^2 - 3^2 = 16$, $h = 4\text{m}$

Area of triangle:

$A = \frac{1}{2}6(4) = 12 \text{ m}^2$

Total area $\approx 14.13 + 48 + 12$

$\approx 74.13 \text{ m}^2$

(Area is in square units ✓)</td>
</tr>
<tr>
<td>Incorrectly substituting values into an algebraic statement</td>
<td>What is the perimeter of a new rectangle if the length is doubled and the width is five times the old width?

$P = 2l + 2w$

Answer: $P = 2l + 5w$</td>
<td>Show the steps when substituting new dimensions into a formula.</td>
<td>New length = $2l$

New width = $5w$

Perimeter of a rectangle:

$P = 2l + 2w$

$P = 2(\ \ ) + 2(\ \ )$

Substitute the new dimensions:

$P = 2(2l) + 2(5w)$

$P = 4l + 10w$, perimeter of enlarged rectangle</td>
</tr>
</table>

ACTIVITY

Composite Figures

PERFORMANCE CRITERIA

- Calculating the perimeter or area of a given composite or regular geometric figure
  - use of the correct formula
  - demonstration of augmentation or breaking down a figure into components
  - accuracy of calculation with correct units
  - validation of the answer
- Using algebra to scale the area or perimeter of geometric figures
  - appropriate and correct identification of the variables
  - use of the correct formula
  - answer presented in its simplest form

CRITICAL THINKING QUESTIONS

1. What are three possible units used for measuring perimeter and for measuring area?

2. What are the basic shapes to look for when either breaking down a figure or augmenting it?

3. How many triangles can be formed from the center of a hexagon using each side as a base? What is the area of each triangle? What is the sum of the areas of the triangles?

4. When would it be reasonable to use variables for the area or perimeter of a figure?

5. What determines which basic shape to use when finding the area of an irregularly shaped geometric figure?

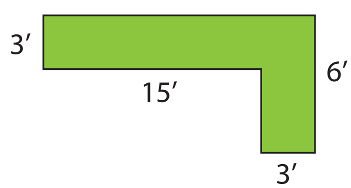
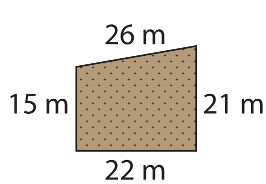
6. How do you know if you have used the best basic shapes to calculate area or perimeter?

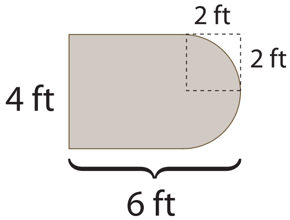
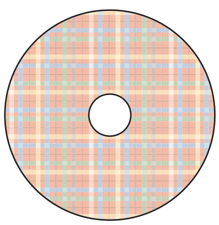
Tips for Success

- Make drawings as accurate and detailed as possible. Do not make assumptions—if a measurement is not given, carefully determine how it can be derived from the information provided.
- The methodology for using geometric formulas from section 3.5 still applies. Identify basic shapes as part of step 1 in the methodology.

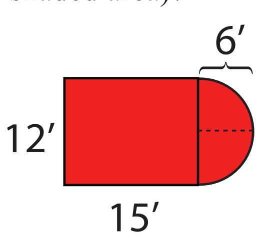
Demonstrate Your Understanding

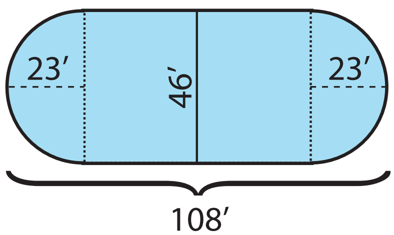
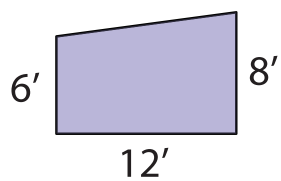
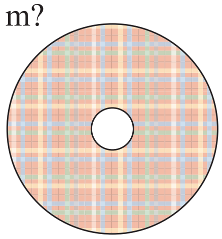
1. Solve the following perimeter problems.

Perimeter Problems	Worked Solution	Validation
a) How much edging is needed for an L-shaped flower bed with the following measurements:  		
b) An electric fence is to be installed on a farmer's plot of land. How much fencing is required if the dimensions are as pictured?  		

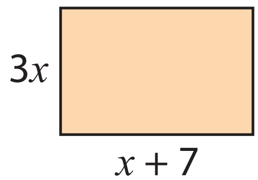
Perimeter Problems	Worked Solution	Validation
c) How much edge trim is needed for a countertop with the following measurements? 		
d) How much ribbon is required to trim a circular skirt along the waist and along the hem if the diameter of the skirt is 2 m and the waist hole has a circumference of 62.8 cm? 		
e) How much molding is needed to trim around a hexagonal bathroom floor if each side is 6 ft long?		

2. Solve the following problems.

Problems	Worked Solution	Validation
a) What is the area of red paint in a basketball key (the shaded area)? 		

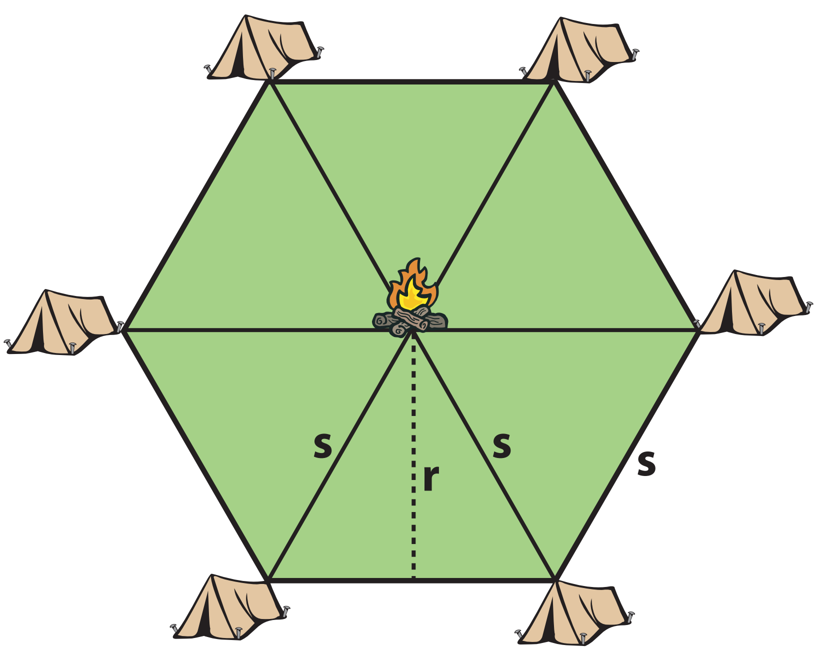
Problems	Worked Solution	Validation
b) If a skating rink has ice 3 inches thick, what volume of ice is there, if the rink dimensions are as follows: 		
c) How many square feet of tiles are needed to cover this bathroom floor? 		
d) How much material is required to make a circular skirt if the waist hole has a circumference of 62.8 cm and the diameter of the skirt is 2 m? 		
e) How many square feet of tiles are needed to floor a hexagonal bathroom if each side is 6 feet long and the radius of the inscribed circle is 5.2 feet?		

3. Solve the following problems in algebraic terms. Simplify your answers.

Algebra Problems	Worked Solution
a) Find the area: 	
b) A circle has diameter of $x + 2$. Write an equation to find the circumference.	
c) A right triangle's height is twice the base. Write an equation for the area.	

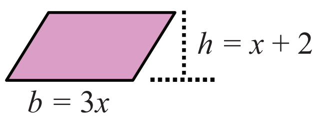
Algebra Problems	Worked Solution
d) Six scouts arrange their campsite so that each tent is equidistant from the next tent **and** from the fire. How much bigger would the area of the campsite be if they reposition the tents so that they are *twice* as far from the fire? Use the graphic below as needed.	
e) How much bigger would the area of the campsite be if, instead of twice the distance, they reposition their tents so that each one is *three times the original distance* from the fire and the adjacent tents? Use the graphic below as needed.	

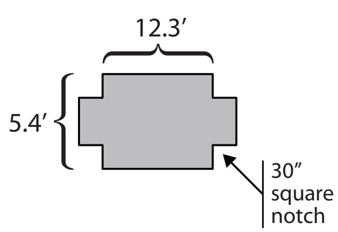
Hint: The height (r) of the triangle formed by two adjacent tents and the fire doubles when the distance from a tent to the fire doubles.

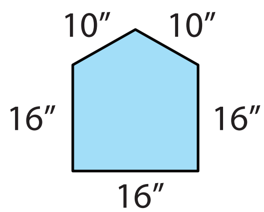
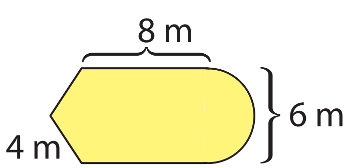


Identify and Correct the Errors

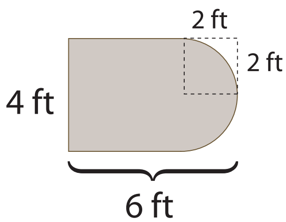
In the second column, identify the error(s) in the worked solution or validate its answer.
If the worked solution is incorrect, solve the problem correctly in the third column and validate your answer.

Worked Solution	Identify Errors or Validate	Correct Process
1) Write an equation for the area:  $A = bh$ $= (3x)(x+2)$		

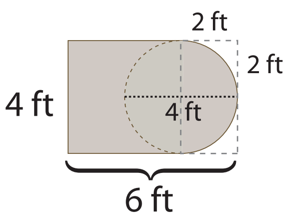
Worked Solution	Identify Errors or Validate	Correct Process
2) Find the area:  $A_L = 12.3' \times 5.4'$ $= 66.42 \text{ ft}^2$ $A = 66.42 - 4 \times 30''$ $A = 66.42 - 120$ $= -53.58$		
	Validation	

Worked Solution	Identify Errors or Validate	Correct Process
3) Find the area of the pentagon below:  $A = (16)^2 + \frac{1}{2}(16 \times 10)$ $= 256 + 80$ $= 336 \text{ in}^2$		
4) Find the area:  Not enough information to work the problem.		

<table>
<tr><th>Worked Solution</th><th>Identify Errors or Validate</th><th>Correct Process</th></tr>
<tr><td>
5) What is the area of a countertop with the following measurements?



Augmentation:



$a = lw$, $r = \frac{1}{2}d$, $a = \pi r^2$
and $A = lw + \frac{1}{2}\pi r^2$
Square feet
Units agree

$A = 6\text{ ft }(4\text{ ft}) + \frac{1}{2}\pi(2\text{ ft})^2$
$A = 24\text{ ft}^2 + 6.28\text{ ft}^2$
$= 30.28\text{ ft}^2$

Validate

- square feet ✓
- 30.28 – 6.28 = 24 ✓
</td><td></td><td></td></tr>
</table>