Our number system has two important sets of numbers: rational and irrational. The most common irrational numbers result from taking the square root of non-perfect squares. The examples below show how square roots are commonly used in different professions.

**Home decorating**
A remnant of carpet in the shape of a square is advertised as 16 square yards. That will cover an area that measures 4 yards by 4 yards or a room that is 12 feet by 12 feet.

**Law enforcement**
In order to find the speed of a car after a collision, an investigator can measure the length of the skid marks (L) and apply the formula: speed = \( 2\sqrt{L} \)

**Utilities**
A device called a Square Root Planimeter is used by oil, gas and utility companies for evaluation of flowmeter charts to average square root recordings of static and differential pressure.

**Computer programming**
Square root and absolute value functions are often used in computer programming to ensure a positive number is used for calculations.

**Distance formula**
Calculation of the distance between two points is done by taking the square root of the sum of the squares of the differences between the \( x \) and \( y \) coordinates.

**Statistics**
Standard deviation is the square root of the variance of data from the mean.

---

**Learning Objectives**

- Simplify expressions containing square roots
- Evaluate and approximate square roots
- Classify numbers as rational or irrational

---

**Terminology**

**Previously Used**
- exponent
- factor
- integer
- ratio

**New Terms to Learn**
- imaginary number
- irrational number
- radical sign
- radical expression
- radicand
- rational number
- rationalizing
- real numbers
- root
- square root
Notation

The symbol for square root is called a **radical sign**: \( \sqrt{} \). The number under the radical sign is the **radicand**. Together they form a **radical expression**.

Another representation for the positive square root of five is \( 5^{\frac{1}{2}} \), where the fraction \( \frac{1}{2} \), written as the **exponent**, represents the radical sign.

**Square Roots**

- A **square root** of a number \( a \) is a number \( r \) such that \( r^2 = a \), or in words, a number \( r \) whose **square** (the result of multiplying the number by itself) is \( a \). For example, 3 is the square root of 9 because \( 3^2 = 9 \).
- The square of a square root is the number itself \( \sqrt{a} \cdot \sqrt{a} = a \). For example, \( \sqrt{3} \cdot \sqrt{3} = 3 \).
- The square root of a number can be represented as either a positive or a negative value:
  - \( \sqrt{49} \) indicates the positive square root of 49 and is equal to 7 because \( 7^2 = 49 \).
  - \( -\sqrt{49} \) indicates the negative square root of 49 and is equal to –7 because \( (-7)^2 = 49 \).
  - \( \pm\sqrt{49} \) means both the positive and negative square roots of 49: 7 and –7.

The square root of a number can be visualized by the Home Decorating example in the introduction. The edge of the carpet is the square root of the area of the carpet.

In general then, \( \sqrt{r^2} = r \) That is, the square root of “r squared” is equal to “r.”

We can extend the concept of square roots with a similar definition. For example a cube root of a number \( a \) is a number \( r \) such that \( r^3 = a \), or in words, a number \( r \) whose **cube** (the result of multiplying the number three times) is \( a \). The cube of a cube root is the number itself:

\[
\sqrt[3]{a} \cdot \sqrt[3]{a} \cdot \sqrt[3]{a} = a
\]

**Square Roots of Negative Numbers**

In the set of real numbers, there is no number whose square is negative; therefore, both square roots of a negative number are undefined within the set of real numbers. The square root of a negative number is called **imaginary** and is indicated with a lower case \( i \); for example, \( \sqrt{-49} = 7i \).

**Radical coefficients**

If there is a number in front of the radical sign (similar to a coefficient), then the radical expression is to be multiplied by that number:

\[
2\sqrt{36} = 2 \times \sqrt{6^2} = 2 \times 6 = 12
\]
Classifying Numbers

The Real Number System has many categories for classifying numbers. You have already worked with whole numbers and integers. We now will introduce two more categories of numbers: Rational and Irrational numbers.

Rational numbers are those numbers that can be expressed as the ratio of two integers. Here are some examples of rational numbers:

\[ 7, \ 5.23, \ \frac{5}{6}, \ \sqrt{36}, \ 0.\overline{333} \]

Why is this the case? We can rewrite each number as the ratio of two integers:

\[ 7 = \frac{7}{1} \quad 5.23 = \frac{523}{100} \quad 36^{\frac{1}{2}} = \frac{6}{1} \quad 0.\overline{333} = \frac{1}{3} \]

Irrational numbers are those numbers that cannot be expressed as the ratio of two integers. The Real numbers include the set of rational numbers as well as the set of irrational numbers. The following are examples of irrational numbers:

\[ \sqrt{7}, \ \pi, 1.202002..., \text{ and } \sqrt{124} \]

\[ \sqrt{7} \] is an irrational number. The value of \( \sqrt{7} \) can be estimated; it is between 2 and 3. The first few digits are: 2.645751311064590590501615753639..., but that is not the exact value; it is a decimal approximation. In order to work with this value, we can determine how many decimal places are accurate enough for our purposes and then round off or truncate the number.

Because rounding off a number leads to less precise calculations and there are an infinite number of decimal places in the approximation of any irrational number, mathematicians prefer to keep irrational number, such as \( \sqrt{7} \), in radical form. This is the exact value, and certainly easier to read, write, say, and even easier to use for calculations than the decimal approximation.

Perfect Square Factors

A perfect square is a number that can be written as the product of another whole number times itself. For example, 16 is a perfect square because it can be written as 4 \( \times \) 4. When finding (often referred to as taking) the square root of a given number, determine if it has any perfect square factors. For example, the perfect square factors of 12 are 1 and 4. While 2, 3 and 6 are also factors of 12, none of the three is a perfect square factor. 1, 4, 25, and 100 are perfect square factors of 100; 2, 5, 20, and 50 are not.

Simplifying Expressions with Square Roots

The following two properties allow simplification of square root expressions.

**The Product Property for Square Roots** states that the product of two square roots equals the square root of their product.

For all non-negative real numbers \( \alpha \) and \( \beta \),

\[ \sqrt{\alpha} \cdot \sqrt{\beta} = \sqrt{\alpha \beta} \quad \text{and} \quad \sqrt{\alpha \beta} = \sqrt{\alpha} \cdot \sqrt{\beta} \]

**Example:** \( \sqrt{4} \cdot \sqrt{3} = \sqrt{12} \) and \( \sqrt{12} = \sqrt{4} \cdot \sqrt{3} \)

**The Quotient Property for Square Roots** states that the quotient of two square roots equals the square root of their quotient.

For all non-negative real numbers \( \alpha \) and \( \beta \), where \( \beta \neq 0 \),

\[ \sqrt{\frac{\alpha}{\beta}} = \frac{\sqrt{\alpha}}{\sqrt{\beta}} \quad \text{and} \quad \frac{\sqrt{\alpha}}{\sqrt{\beta}} = \sqrt{\frac{\alpha}{\beta}} \]

**Example:** \( \sqrt{18} \cdot \sqrt{2} = \sqrt{36} \) and \( \sqrt{\frac{18}{2}} = \sqrt{\frac{18}{2}} \)
## Methodologies

### Simplifying Expressions with Square Roots

A simplified expression with radical signs must meet the following criteria:

1. No perfect squares in the radicand
2. No fractions in the radicand
3. No square root expressions in the denominator

#### Example 1:
Simplify: \( \sqrt{200x^2} \cdot \sqrt{2y^2} \)

#### Example 2:
Simplify: \( \sqrt{30x} \cdot \sqrt{25x^3} \)

---

**Steps in the Methodology**

<table>
<thead>
<tr>
<th>Steps in the Methodology</th>
<th>Example 1</th>
<th>Example 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 1</strong> Set up the problem</td>
<td>Visually align the problem so you can see the relationships to determine if the problem is to be simplified, multiplied, or divided.</td>
<td>( \sqrt{200x^2} \cdot \sqrt{2y^2} )</td>
</tr>
<tr>
<td><strong>Step 2</strong> Write under one radical sign</td>
<td>Use the appropriate product and/or quotient properties to rewrite the problem so that there is only one radical sign.</td>
<td>( \sqrt{200 \cdot x^2 \cdot 2 \cdot y^2} )</td>
</tr>
<tr>
<td><strong>Step 3</strong> Simplify</td>
<td>Multiply, divide, or reduce fractions as needed to simplify the radicand.</td>
<td>( \sqrt{400 \cdot x^2 \cdot y^2} )</td>
</tr>
<tr>
<td><strong>Step 4</strong> Find perfect square factors</td>
<td>Rewrite the radicand using its perfect square factors. If there is a denominator, make sure it is a perfect square.</td>
<td>( \sqrt{20^2 \cdot x^2 \cdot y^2} ) no denominator</td>
</tr>
<tr>
<td><strong>Special Case:</strong></td>
<td>If the denominator is not a perfect square, see Model 2 and the accompanying Technique.</td>
<td></td>
</tr>
<tr>
<td><strong>Step 5</strong> Find square roots</td>
<td>Using the Product or Quotient Properties for Square Roots, take the square root of each perfect square factor found in step 4. If there are factors that are not perfect squares, leave them under the radical sign.</td>
<td>20xy</td>
</tr>
<tr>
<td><strong>Step 6</strong> Validate</td>
<td>Assign a value for each variable and determine if the evaluation of the original problem is equal to the evaluation of the answer. You may use a calculator. See Technique 1.</td>
<td>Let ( x = 3 ) and ( y = 4 )</td>
</tr>
<tr>
<td><strong>Note:</strong></td>
<td>Do not use 0, 1, or 2 as values for the variables.</td>
<td>( \sqrt{200 \cdot 3^2} \cdot \sqrt{2 \cdot 4^2} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>= ( \sqrt{1800} \cdot 32 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>= 240</td>
</tr>
<tr>
<td></td>
<td></td>
<td>and</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20 ( \cdot 3 \cdot 4 = 240 ) ✓</td>
</tr>
</tbody>
</table>
Models

Model 1: Dividing Radical Expressions

Simplify: \( \frac{\sqrt{5x^2}}{\sqrt{125x}} \)

Step 1  Simplify \( \frac{\sqrt{5x^2}}{\sqrt{125x}} \)

Step 2  Write under one radical \( \frac{\sqrt{5x^2}}{\sqrt{125x}} \)

Step 3  Simplify \( \frac{\sqrt{5} \cdot x}{\sqrt{125 \cdot x}} = \frac{x}{\sqrt{25}} \)

Step 4  Find perfect square factors 25 is a perfect square: \( \sqrt{\frac{x}{5^2}} \)

Step 5  Find square roots  Answer: \( \frac{\sqrt{x}}{5} \)

Step 6  Validate  Let \( x = 3 \)

\[
\frac{\sqrt{5 \cdot 3^2}}{\sqrt{125 \cdot 3}} = \frac{\sqrt{45}}{\sqrt{375}}
\]

\[
= \frac{6.708}{19.365} \approx 0.346
\]

and \( \frac{\sqrt{3}}{5} \approx 0.346 \)

Techniques

Evaluating Square Roots

First, mentally simplify an expression. Then, using a computational tool, determine the decimal value of the radicand and calculate the product of the coefficient(s) and the decimal approximation of the radical value.

Round off to the desired number of places. Use the symbol “≈” to mean “approximately equal.”

Find the decimal approximation of \( 3\sqrt{7} \) to thousandths: \( 3\sqrt{7} \approx 7.937 \).
### Rationalizing a Denominator

When dividing rational expressions, the best case is that the denominator is a perfect square. But what if it is not? We can use a technique called **rationalizing** in order to turn the denominator into a perfect square so that we can continue with Step 4 of the Methodology.

Here is a simple example of how to rationalize a denominator:

\[
\sqrt{\frac{x}{3}} = \sqrt{\frac{x \cdot 3}{3 \cdot 3}} = \sqrt{\frac{3x}{9}}
\]

The starting expression does not contain a perfect square in the denominator. Identify what number, when multiplied by the denominator, creates a perfect square.

Think, "What number, when multiplied by 3, will create a perfect square?"

The answer is 3 because \(3 \times 3 = 9\), a perfect square.

Multiply both the numerator and denominator by this number. Recall that multiplying a fraction by 1 in the form of \(\frac{\text{any number}}{\text{the same number}}\) does not change its value (Identity for Multiplication Property).

Once we have carried out the multiplication, we have the end expression that contains a perfect square in the denominator. We can now continue with Step 4 of the Methodology.

### Model 2: Simplifying when the Denominator is not a Perfect Square

Simplify: \(\frac{\sqrt{5x^2}}{\sqrt{250x}}\)

**Step 1** Simplify \(\frac{\sqrt{5x^2}}{\sqrt{250x}}\)

**Step 2** Write under one radical \(\frac{\sqrt{5x^2}}{\sqrt{250x}}\)

**Step 3** Simplify \(\frac{\sqrt{5x^2}}{\sqrt{250x}} = \sqrt{\frac{x}{50}}\)

The denominator is not a perfect square. Use the Technique for Rationalizing a Denominator:

Think, “What number, when multiplied by 50, will create a perfect square?” The answer is 2.

\[
\sqrt{\frac{x \cdot 2}{50 \cdot 2}} = \sqrt{\frac{2x}{100}}
\]
Evaluate or approximate the following square roots to four decimal places if needed.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Evaluate</th>
<th>Validate</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sqrt{121})</td>
<td>11</td>
<td>11 (\times) 11 = 121 ✓</td>
</tr>
<tr>
<td>(\sqrt{42})</td>
<td>(\approx 6.4807)</td>
<td>The square root was determined by using a calculator. To validate, take the approximate answer and square it. The answer should be close to the original radicand. (6.4807^2 \approx 41.9995) ✓</td>
</tr>
<tr>
<td>(-\sqrt{9})</td>
<td>(-3)</td>
<td>((-3)(-3) = 9) ✓</td>
</tr>
<tr>
<td>(\sqrt{10})</td>
<td>(\approx 3.1623)</td>
<td>The square root was determined by using a calculator. To validate, take the approximate answer and square it. The answer should be close to the original radicand. (3.1623^2 \approx 10.0001) ✓</td>
</tr>
<tr>
<td>(\sqrt{24})</td>
<td>(\approx 4.8990)</td>
<td>Exact answer: (\sqrt{24} = \sqrt{4 \cdot 6} = 2 \cdot 2)</td>
</tr>
<tr>
<td> </td>
<td> </td>
<td>Approximate answer: (\approx 2 \times 2.4495 \approx 4.8990) ✓</td>
</tr>
<tr>
<td>(\sqrt{\frac{9}{25}})</td>
<td>(\frac{\sqrt{9}}{\sqrt{25}} = \frac{3}{5})</td>
<td>(\frac{3}{5} \cdot \frac{3}{5} = \frac{9}{25}) ✓</td>
</tr>
<tr>
<td>(\frac{\sqrt{20}}{\sqrt{5}})</td>
<td>(\frac{\sqrt{20}}{\sqrt{5}} = \frac{\sqrt{4 \cdot 5}}{\sqrt{5}} = \frac{\sqrt{20}}{\sqrt{5}} = \sqrt{4} = 2)</td>
<td>Use a calculator: (\frac{\sqrt{20}}{\sqrt{5}} = \frac{20}{5} = \sqrt{4} = 2) ✓</td>
</tr>
<tr>
<td>(\sqrt{a^3})</td>
<td>(\sqrt{a^3} = \sqrt{a \cdot a \cdot a} = \sqrt{a^2 \cdot a} = \sqrt{a^2} \cdot \sqrt{a} = a\sqrt{a})</td>
<td>Let (a = 4)</td>
</tr>
<tr>
<td> </td>
<td> </td>
<td>(\sqrt{4^3} = \sqrt{64} = 8)</td>
</tr>
<tr>
<td> </td>
<td> </td>
<td>(and)</td>
</tr>
<tr>
<td> </td>
<td> </td>
<td>(4\sqrt{4} = 4 \cdot 2 = 8) ✓</td>
</tr>
</tbody>
</table>
## Determining if a Number is Rational

Find a representation of a given number as a ratio of two integers. If you can, the number is rational. If not, it is irrational.

### Model 4

Classify the following numbers as rational, irrational, or neither. Give your reasoning and then evaluate, if necessary.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Classification</th>
<th>Reasoning</th>
<th>Evaluate</th>
</tr>
</thead>
<tbody>
<tr>
<td>2/3</td>
<td>rational</td>
<td>It is a fraction—a ratio of two integers: ( \frac{2}{3} ).</td>
<td>2/3</td>
</tr>
<tr>
<td>23</td>
<td>rational</td>
<td>23 can be expressed as a fraction: ( \frac{23}{1} ).</td>
<td>23</td>
</tr>
<tr>
<td>5.25</td>
<td>rational</td>
<td>It can be expressed as a ratio: 525/100.</td>
<td>5.25</td>
</tr>
<tr>
<td>( \sqrt{36} )</td>
<td>rational</td>
<td>It simplifies to a whole number.</td>
<td>6</td>
</tr>
<tr>
<td>( -\sqrt{36} )</td>
<td>rational</td>
<td>( -\sqrt{36} = -6 ) or ( \frac{-6}{1} ).</td>
<td>(-6)</td>
</tr>
<tr>
<td>( \sqrt{6} )</td>
<td>irrational</td>
<td>6 is not a perfect square, nor does it have a perfect square factor to take the root of. It cannot be represented by a ratio of integers.</td>
<td>( \approx 2.449 ) to the nearest thousandth</td>
</tr>
<tr>
<td>4.10100100010000...</td>
<td>irrational</td>
<td>It is a non-repeating, non-terminating decimal.</td>
<td>( \approx 4.1 ) to the nearest tenth</td>
</tr>
<tr>
<td>( \sqrt{-36} )</td>
<td>neither</td>
<td>The square root of a negative number is an imaginary number.</td>
<td>( 6i )</td>
</tr>
</tbody>
</table>
## Addressing Common Errors

<table>
<thead>
<tr>
<th>Issue</th>
<th>Incorrect Process</th>
<th>Resolution</th>
<th>Correct Process</th>
<th>Validation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forgetting to remove the radical sign when taking the square root</td>
<td>( \sqrt{9x^2} = \sqrt{3x} )</td>
<td>Squaring and taking the square root of a number are opposite processes. Taking the square root of a perfect square removes both the radical and the exponent.</td>
<td>( \sqrt{9x^2} = 3x )</td>
<td>Enter any value for the variable and compare expressions. Let ( x = 3 ); compare: ( \sqrt{9 \cdot 3^2} = \sqrt{81} = 9 ) and ( 3 \cdot 3 = 9 \ ✓ )</td>
</tr>
<tr>
<td>Not removing all perfect square factors when simplifying a radical expression</td>
<td>( \sqrt{48} = \sqrt{4 \cdot 12} = 2\sqrt{12} )</td>
<td>Write the radicand as a product of prime factors under the radical. Pair the prime factors into perfect squares.</td>
<td><strong>Factor:</strong> ( 48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 ) ( \sqrt{48} = \sqrt{2^2 \cdot 2^2 \cdot 3} = 2 \cdot 2\sqrt{3} = 4\sqrt{3} )</td>
<td>Work backwards: ( 4\sqrt{3} \cdot 4\sqrt{3} = 16 \cdot 3 = 48 \ ✓ )</td>
</tr>
<tr>
<td>Simplifying radical fractions without using the Quotient Property for square roots</td>
<td>( \frac{\sqrt{6x^2}}{\sqrt{50xy^2}} = \frac{x\sqrt{6}}{5y\sqrt{2x}} )</td>
<td>Use the Quotient Property for square roots to determine if the fractions reduce to perfect square factors.</td>
<td>( \frac{\sqrt{6x^2}}{\sqrt{50xy^2}} = \frac{\sqrt{6x^2} \cdot \sqrt{25y^2}}{\sqrt{50xy^2} \cdot \sqrt{25y^2}} = \frac{\sqrt{2} \cdot 3\sqrt{x}}{\sqrt{2} \cdot 5\sqrt{xy^2}} = \frac{\sqrt{3x}}{\sqrt{5y}} ) ( \frac{\sqrt{3} \cdot \sqrt{3}}{\sqrt{5} \cdot \sqrt{4}} = \frac{3}{20} = 0.15 \ ✓ )</td>
<td>Let ( x = 3 ) and ( y = 4 ): ( \sqrt{\frac{6x^2}{50xy^2}} = \sqrt{\frac{6 \cdot 3^2}{50 \cdot 3 \cdot 4^2}} = \frac{\sqrt{54}}{\sqrt{2400}} = \frac{\sqrt{0.0225}}{0.15} = 0.15 ) and ( \frac{\sqrt{3} \cdot \sqrt{3}}{\sqrt{5} \cdot \sqrt{4}} = \frac{3}{20} = 0.15 \ ✓ )</td>
</tr>
<tr>
<td>Not knowing when to remove the radical sign</td>
<td>( \sqrt{81} = \sqrt{9} = \sqrt{3} ) or ( \sqrt{81} = \sqrt{9} = 3 )</td>
<td>Once the square root has been taken, the radical sign is eliminated.</td>
<td>( \sqrt{81} = \sqrt{3 \cdot 3 \cdot 3 \cdot 3} ) ( = \sqrt{3^2 \cdot 3^2} ) ( = 3 \cdot 3 = 9 )</td>
<td>( 9 \cdot 9 = 81 \ ✓ )</td>
</tr>
</tbody>
</table>
Before proceeding, you should have an understanding of each of the following:

- Relating perfect squares and square roots
- Using the methodology to apply the Product and Quotient Properties for Square Roots to simplify radical expressions
- Approximating the value of a square root with a calculator
- Classifying a number as rational or irrational
Performance Criteria

- Simplifying any given radical expression
  - application of the appropriate property or properties
  - accuracy of calculations
  - validation of the answer

- Classifying a number as rational or irrational
  - correctly classified
  - supported by justification

- Evaluating or approximating the square root of a number using a calculator
  - accurately and appropriately rounded off
  - appropriate use of = and ≈
  - validation of the answer

Critical Thinking Questions

1. How many square roots of any given number are there?

2. How do you know if a radical expression is in its simplest form?

3. How do you show the exact answer or an approximate answer to a square root problem?
4. What makes a number rational?

5. What is the difference between “$\sqrt{36} =$” and “find the square roots of 36”?

6. How do you find perfect square factors when simplifying a radical expression?

7. What is the difference between “the negative square root of a number” and “the square root of a negative number”?

**TIPS FOR SUCCESS**

- Validate your answer by using your calculator
- Document each step to avoid shortcuts that lead to errors
- The square root radical sign “undoes” the square of a number: $\sqrt{3^2} = 3$
1. Simplify the following radical expressions:

<table>
<thead>
<tr>
<th>Problem</th>
<th>Answer</th>
<th>Validation</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) (\sqrt{150})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b) (-\sqrt{25y^4})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c) (-\sqrt{900})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d) (\sqrt{54x^2})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>e) (\sqrt{20x^2})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f) (\sqrt{48x^3})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>g) (\sqrt{12x^2})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>h) (\sqrt{50x})</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2. Use the Product and Quotient Properties to simplify the following radical expressions:

<table>
<thead>
<tr>
<th>Problem</th>
<th>Answer</th>
<th>Validation</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) (\sqrt{9} \cdot \sqrt{36})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b) (\sqrt{100} \cdot \sqrt{25})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c) (\sqrt{5a} \cdot \sqrt{20a^3})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d) (\frac{\sqrt{100}}{\sqrt{25}})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>e) (\frac{\sqrt{36}}{\sqrt{4}})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f) (\frac{\sqrt{8y^5}}{\sqrt{2y}})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>g) (\frac{\sqrt{3y^2}}{\sqrt{18y}})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>h) (\frac{\sqrt{5a^3}}{\sqrt{7a^2}})</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3. Use your calculator to evaluate or approximate the following square roots. Use four decimal places for approximate answers.

a) \( \sqrt{121} \)  
   \text{answer: } 

b) \( \sqrt{42} \)  
   \text{answer: } 

c) \( -\sqrt{9} \)  
   \text{answer: } 

d) \( \sqrt{10} \)  
   \text{answer: } 

e) \( -\sqrt{64} \)  
   \text{answer: } 

f) \( -\sqrt{19} \)  
   \text{answer: } 

g) \( \sqrt{1000} \)  
   \text{answer: } 

h) \( \sqrt{2} \)  
   \text{answer: } 

4. Identify the following as rational or irrational and give a justification:

<table>
<thead>
<tr>
<th>Problem</th>
<th>Rational or Irrational?</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) ( \sqrt{36} )</td>
<td>Rational</td>
<td>( \sqrt{36} = 6 ) is a perfect square.</td>
</tr>
<tr>
<td>b) ( \sqrt{7} )</td>
<td>Irrational</td>
<td>( \sqrt{7} ) is an irrational number.</td>
</tr>
<tr>
<td>c) ( 3\sqrt{16} )</td>
<td>Rational</td>
<td>( 3\sqrt{16} = 12 ) is an integer.</td>
</tr>
<tr>
<td>d) ( 2\sqrt{6} )</td>
<td>Irrational</td>
<td>( 2\sqrt{6} ) is an irrational number.</td>
</tr>
<tr>
<td>e) ( 0.2\overline{22} )</td>
<td>Rational</td>
<td>( 0.2\overline{22} ) is a repeating decimal.</td>
</tr>
<tr>
<td>f) ( 3 )</td>
<td>Rational</td>
<td>( 3 ) is an integer.</td>
</tr>
<tr>
<td>g) ( 1.010010001... )</td>
<td>Irrational</td>
<td>( 1.010010001... ) is a non-terminating, non-repeating decimal.</td>
</tr>
</tbody>
</table>
In the second column, identify the error(s) in the worked solution or validate its answer. If the worked solution is incorrect, solve the problem correctly in the third column and validate your answer.

<table>
<thead>
<tr>
<th>Worked Solution</th>
<th>Identify Errors or Validate</th>
<th>Correct Process</th>
<th>Validation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) $-\sqrt{9} = \sqrt{-9}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) $\sqrt{42}$ is simplified because $\sqrt{42} = \sqrt{6 \cdot 7}$, so there are no perfect square factors to remove from the radicand.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) $\sqrt{120} = \sqrt{12 \cdot 10}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4) $\sqrt{16} = \sqrt{4} = 2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5) $\frac{\sqrt{36x^2}}{\sqrt{12x}} = \frac{6x}{\sqrt{12x}}$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>